

# Finite-Difference Performance Analysis of Jet Pumps

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## Theme

**T**RADITIONALLY jet pump design<sup>1,2</sup> has been based on experiment rather than theory, and, what theory there is has been developed from simple energy and momentum balances. The present work presents a more exhaustive analysis capable of application to a much broader class of problems. It is based on a new primitive-variable finite-difference procedure,<sup>3</sup> suitably modified to predict two-dimensional axisymmetric jet pump flows.<sup>4</sup> The analysis permits the investigation of various parameters and their effect on overall performance, and in so doing explains the mechanism of mixing between the primary and secondary fluids. Computed values for both the internal flow characteristics and the overall performance of various jet pump configurations are presented in detail in the full-length paper, together with recent confirmatory experimental data.

## Contents

### Analysis

The technique discussed here simulates two-dimensional axisymmetric jet pumps directly in terms of a two-equation  $k$ - $\epsilon$  turbulence model,<sup>5</sup> incorporated in the governing elliptic partial differential equations, and solution is directly by a finite-difference relaxation technique. Its improvement and use will significantly reduce the cost and time required for jet pump design. Because it possesses many advantages, the present work is developed directly on an approach which focuses attention directly on the primitive pressure-velocity variables, the basic ideas of which are available in the 1974 Imperial College TEACH computer program.<sup>3</sup> Elliptic partial differential equations for axial velocity  $u$ , radial velocity  $v$ , kinetic energy of turbulence  $k$ , and turbulence length scale  $\ell$  are all taken in a common form, the equations differing primarily in their diffusion coefficients and final source terms. Interlinkages between the equations present difficulties to solution. Those between the axial and radial velocity components are of a peculiar kind, each containing an unknown pressure gradient and the components are linked additionally by another equation, that of mass conservation, in which pressure  $p$  does not appear.

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Index categories: Jet, Wakes, and Viscid-Inviscid Flows Interactions; Nozzle and Channel Flow; Viscous Nonboundary-Layer Flows.

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The work here incorporates the following: a) a finite-difference procedure is used in which the dependent variables are the velocity components and pressure; b) the pressure correction  $p'$  is deduced from an equation which is obtained by the combination of the continuity equation and the momenta equations (yielding a new form of what is known in the literature as the Poisson equation for pressure); c) the idea is present at each iteration of a first approximation  $u^*$ ,  $v^*$ , and  $p^*$  to the solution followed by a succeeding correction  $u = u^* + u'$ ,  $v = v^* + v'$  and  $p = p^* + p'$  ( $u'$  and  $v'$  are related to  $p'$ ); d) the procedure incorporates displaced grids for the axial and radial velocities  $u$  and  $v$ , which are placed between the nodes where pressure  $p$  and other variables are stored; and e) an implicit line-by-line relaxation technique is employed in the solution procedure (requiring a tridiagonal matrix to be inverted in order to update a variable at all points along a column).

Since they are readily available elsewhere,<sup>3,4,6</sup> no further details of the simulation and computational method need be included here.

## Results and Discussion

The computer program has been set up to make predictions for the same configuration as studied experimentally and described in the full paper. The predictions shown relate to an area ratio  $R$  = driving nozzle area/mixing tube area of 0.25 for different primary jet velocities  $u_{m0}$  and nozzle axial positions with respect to the mixing tube  $1/D$  where 1 is the distance from the primary nozzle to mixing tube and  $D$  is the mixing tube diameter. As examples of the current predictive capability of the program, several sample computations are presented to illustrate: a) the internal behavior of the jet pump, particularly in the region of mixing between the primary and secondary flows, and the effect of nozzle axial position; and b) the predicted overall performance of the jet pump and a comparison with experimental results.

Computations were generally made with a  $15 \times 15$  variable size grid, which allowed the solution of a typical problem to be obtained in about 100-150 iterations, and taking the equivalent of about 1-2 min of CDC6600 CP time.

Discussed first is a configuration with primary nozzle to mixer tube spacing  $1/D = 2.25$  and inlet primary jet velocity  $u_{m0} = 22 \text{ msec}^{-1}$ . Figure 1 shows the turbulence energy  $k$  and length scale  $\ell$  distributions within the flow. The energy contours reveal that, at the point of "impact" of the primary jet on the entrained flow, there is a high rate of energy generation. The high values of length scale in this region quickly dissipate the energy into the entrained flow. The rapid decrease in the downstream level of length scale results in a very gradual reduction in energy level in the mixing tube section. This indicates that the mixing tube length is perhaps too great since minimal mixing is actually occurring there. It is also revealing to examine the axial velocity profiles in the mixing region. Figure 2 shows the increase in axial velocity as the entrained fluid nears the primary jet at a section just downstream of the nozzle exit. The collision between the

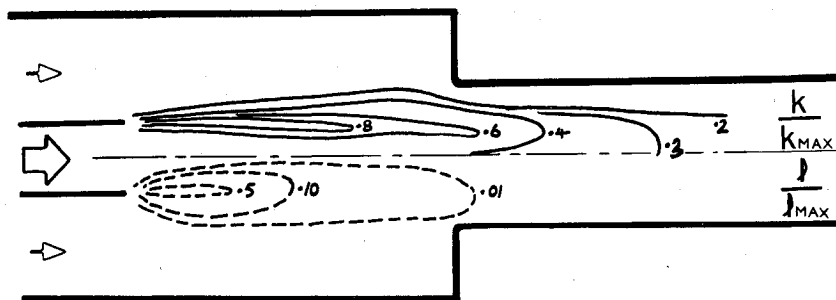


Fig. 1 Predicted turbulence energy and length scale distributions ( $1/D = 2.25$ ,  $u_{mo} = 22 \text{ msec}^{-1}$ ).

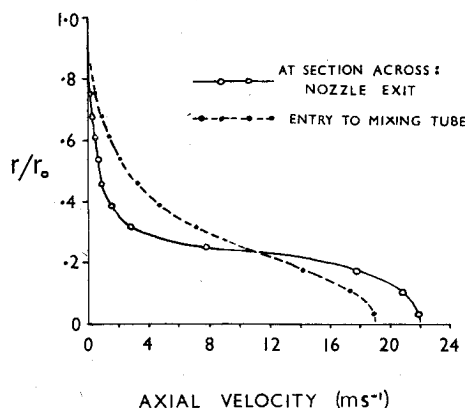


Fig. 2 Predicted axial velocity distributions in mixing region ( $1/D = 2.25$ ,  $u_{mo} = 22 \text{ msec}^{-1}$ ).

radial and axial flows results in a violent mixing process and the consequent generation of turbulence energy. Figures 1 and 2 also show a rapid decay in turbulence energy and axial velocity away from the initial mixing region, particularly in the radial direction outwards toward the suction chamber wall: a fact which suggests that the suction chamber is too large, and that a similar performance would be available with smaller equipment. The position of the nozzle relative to the mouth of the mixing tube is another design feature which the model can easily simulate.

Computations were also made for a second system, differing from the one previously described in only one respect: the nozzle-mixing tube spacing was reduced to  $1/D = 0.5$ . For this case where the nozzle is adjacent to the mouth of the mixing tube, the turbulence energy contours showed that there is less dissipation into the secondary fluid and consequently less entrainment. In terms of predicted entrainment ratio  $M = \text{entrained fluid rate/primary fluid rate}$  it reduced to 0.48 as opposed to the previous system value of 0.58. Naturally intermediate values of the nozzle to mixing tube spacing can be tried to optimize the entrainment ratio. As a further illustration of the predictive capability of the model, the entrainment ratios derived from experiment and theory for a

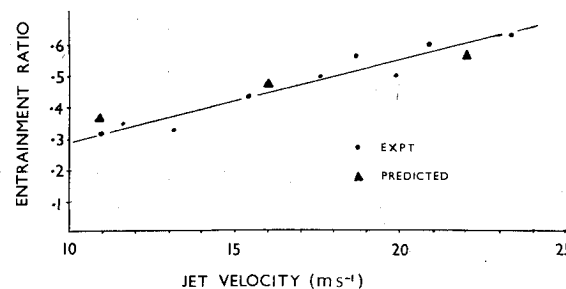


Fig. 3 Comparison of predicted and experimental entrainment ratios ( $1/D = 2.25$ ).

number of primary jet velocities are compared in Fig. 3, for the  $1/D = 2.25$  case. While the actual values (both experiment and theory) are not as high as commercially available pumps (because of relatively low operating pressures), there is good correlation between the predicted and experimental values. Results suggest that a useful design tool is now becoming available, and that further development and application will provide a valuable supplementary technique in practical design situations.

## References

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